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A SIMPLE MODEL OF FIRM BEHAVIOR UNDER REGULATION AND UNCERTAINTY

Stewart C. Myers

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A SIMPLE MODEL OF FIRM BEHAVIOR UNDER REGULATION AND UNCERTAINTY

Stewart C. Myers*

Introduction

It is not new to say that risk is important in regulation. The legal principle guiding rate of return regulation provides that "the return to the equity owner should be commensurate with returns on investments in other enterprises having corresponding risks."[1] This idea, that risk determines allowable return, is incorporated in court and regulatory decisions dating back to the turn of the century.[2] I doubt there is one regulatory case in which the firm's representatives have not argued that the firm is facing high risks and therefore should be allowed a high return, or in which the firm's opponents have not argued the opposite.

With so much talk, one would expect the economic theory of regulation to include a discussion of how a utility's risk is related to demand and cost conditions and to regulatory procedures. But as far as I know this paper is the first attack on this problem.

The paper's line of attack is (1) to calculate the equilibrium investment and output decisions of a monopolistic firm acting under uncertainty, (2) to compare this equilibrium result with the one that would be achieved under competitive conditions and (3) to show how regulatory constraints can affect the behavior of such a firm. The analysis is

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conducted for an extremely simple case with the help of a geometric presentation and a numerical example. I believe this case captures the essential elements of the problem. However, I am not at this time attempting to present a general model or to state theorems about optimal regulatory strategies.

Framework for the Analysis

The analysis will employ the "time-state-preference" framework for analysis of behavior under uncertainty.[3] That is, uncertainty about conditions in a future period is described by specifying a set of possible "states of nature," only one of which can actually occur. The realized values of random variables are contingent on which state of nature occurs.

I will analyze a one-period world, in which there are only two possible future states of nature ($s = 1, 2$). Decisions are made now, at $t=0$, without knowing which state will occur at $t = 1$. (The generalization to many states is trivial; the generalization to many time periods is not.)

Thus, we can describe uncertainty about any relevant variable (cost, demand, profit, production capacity, etc.) by listing the two possible contingent values for the variable. Suppose, for example, that an investment will return $1,000 if $s = 1$ and $1,500 if $s = 2$. A complete specification of the investment's risk-return characteristics is simply the vector [1000,1500].

Under ideal conditions, markets will arise for payments contingent on the occurrence of particular states of nature. For example, we might find a contract, promising payment of $1 if, and only if, state s occurs, selling now for V(1) dollars. V(1) is thus the present value of

a dollar delivered contingent on state 1. In the two-period case, we might observe that $V(1) = \$.50$ and $V(2) = \$.40$. Thus, if an investor wants to purchase one dollar delivered in each state, it will cost him $\$.50 + .40 = \$.90$.

If there are no restrictions on trading in these contingent contracts then the possibility of arbitrage insures that the present market value of any asset is given by

$$PV = V(1)\begin{bmatrix}\text{cash payoff}\\ \text{if } s = 1\end{bmatrix} + V(2)\begin{bmatrix}\text{cash payoff}\\ \text{if } s = 2\end{bmatrix}$$

With $V(1) = \$.50$ and $V(2) = \$.40$ the value of the investment offering [1000, 1500] is

$$PV = .5(1000) + .4(1500) = 1100.$$

If an investment of $1000 is required, the net present value of the asset is

$$NPV = .5(1000) + .4(1500) - 1000 = +100.$$

The prices $V(s)$ also specify the risk-free rate of interest, R_F. Imagine a contract offering a certain return of \$1 -- that is, \$1 contingent on $s = 1$ and \$1 contingent on $s = 2$. The present market value of the contract will be $V(1) + V(2) = .50 + .40 = \$.90$. The implied risk-free rate is

$$.90 = \frac{1}{1 + R_F}$$

$$R_F \cong .11.$$

The Basic Model

Consider a firm which has to invest now (t = 0) in order to build capacity for production in the future (t = 1). Cost and demand conditions for t = 1 are not known. They depend on which of the two possible states of nature occurs.

We will employ the following notation, where s indexes the possible states:

$V(s)$ = present value of \$1 delivered if and only if state s occurs,

$Q(s)$ = the firm's output in s,

Q_M = maximum output, determined by investment at t = 0,

$C(s)$ = cost per unit of output in s,

$P(s)$ = price per unit of output in s, and

$F(Q_M)$ = investment required to build the capacity Q_M.

The firm's objective is to maximize the net present value of its investment, subject to the constraint that, regardless of the state of nature occuring, it cannot produce more than its capacity. Formally, its problem[4] is to:

$$\underset{Q(s),\, Q_M}{\text{Max}} \quad \psi = \sum_{s=1,2} V(s)[P(s) - C(s)]Q(s) - F(Q_M) \tag{1}$$

$$\text{S.T.} \quad \emptyset(s) = Q(s) - Q_M \leqq 0.$$

Assuming Q(1), Q(2), and Q_M all are positive, the conditions for the maximum are:

$$Q(1): \quad V(1)\left[P(1) + Q(1)\frac{\delta P(1)}{\delta Q(1)} - C(1) - Q(1)\frac{\delta C(1)}{\delta Q(1)}\right] - \lambda(1) = 0 \qquad (2)$$

$$Q(2): \quad V(2)\left[P(2) + Q(2)\frac{\delta P(2)}{\delta Q(2)} - C(2) - Q(2)\frac{\delta C(2)}{\delta Q(2)}\right] - \lambda(2) = 0$$

$$Q_M: \quad -\frac{\delta F}{\delta Q_M} + \lambda(1) + \lambda(2) = 0$$

Also: $\emptyset(s)\lambda(s) = 0, \qquad s = 1, 2.$

The conditions are easy to interpret. The shadow prices $\lambda(1)$ and $\lambda(2)$ represent the present value of the difference between marginal revenue and marginal cost. Marginal revenue and marginal cost are not equal unless output is less than capacity, in which case $\emptyset(s) < 0$ and $\lambda(s) = 0$. But the condition for Q_M assures that capacity will limit output in at least one state, since $\delta F/\delta Q_M > 0$. Thus the sum of the shadow prices $\lambda(s)$ also represents the present value of extra capacity, and the condition on Q_M simply states that capacity should be expanded until the marginal cost of an extra unit equals the present value of an extra unit available for use in the various states.

The solution for competitive markets is different in two respects. First, from the viewpoint of the competitive firm, $Q(s)\frac{\delta P(s)}{\delta Q(s)} = 0$ for all s. That is, P(s) appears instead of marginal revenue in the conditions on Q(s). Second, there will entry into, or exit from, the industry until $\Psi = 0$.

There is no reason to expect $\lambda(1)$ to equal $\lambda(2)$ in either the competitive or monopolistic case. This is important, because the λ's reflect the profitability that the utility will enjoy after the true state of nature is revealed -- in other words, they reflect "ex post" profitability. Even in competitive equilibrium, firms may

enjoy "monopolistic profits" or incur substantial losses after the fact. Clearly, the attempt by regulators to impose a "reasonable" or "fair" rate of return after the fact may rule out any chance of approximating the competitive solution.

Regulation as a Substitute for Competition

It is a bit artificial to think of a "competitive solution" in a regulated industry, in which competition is almost by definition imperfect. Nevertheless, regulation has been conceived of as a substitute for competition -- that is, as an attempt to enjoy the welfare-maximizing properties of competitive equilibrium despite the existence of a naturally monopolistic or oligopolistic industry.

Thus in the present context ideal regulation would meet two conditions:

Condition 1. -- Regulation would force the utility to act as if marginal revenue in s equalled $P(s)$ rather than $P(s) + Q(s)\frac{\delta P(s)}{\delta Q(s)}$.

Condition 2. -- If satisfaction of condition 1 leads the firm to a solution where ψ is not zero, then the regulators would impose a lump-sum tax or subsidy[5] equal to $-\psi$.

The hard part is clearly to achieve condition 1. The second condition will not be discussed further in this paper. Regulation will be said to lead to the "competitive solution" if the first condition is satisfied.

Rate of Return Regulation

There are a variety of regulatory strategies that might lead to a competitive solution. The impact of these strategies can be analyzed by adding constraints and observing the changes in Q(1), Q(2) and Q_M.

In real life the constraint is on book rate of return -- i.e., on the ratio of accounting income to the book value of past investment. Presumably a utility's investment decision and operating plans take account of the fact that regulators will force it to lower the price of its product if its ex post rate of return would otherwise exceed some maximum R^*.

Of course the firm does not know what the future value of R^* will be, although an established utility probably can make a good guess. Nor does it know how promptly and effectively the constraint will be enforced. However, I will ignore these complications here. R^* will be assumed known ex ante, and it will be assumed that the constraint is strictly enforced.

Thus, the constraints change to

$$\begin{aligned} \emptyset_A(s) &= Q(s) - Q_M \leq 0, \\ \emptyset_B(s) &= Q(s)[P(s) - C(s)] - (1 + R^*)\, F(Q_M) \leq 0. \end{aligned} \qquad (1a)$$

To simplify notation, let MP(s), "marginal profit" in s, be given by

$$MP(s) = P(s) + Q(s)\frac{\delta P(s)}{\delta Q(s)} - C(s) - Q(s)\frac{\delta C(s)}{\delta Q(s)} .$$

Then the addition of the constraints $\emptyset_B(s)$ changes the conditions for the maximum to:

$$\begin{aligned} Q(s)&:\quad V(s)MP(s) - \lambda_A(s) - \lambda_B(s)[P(s) - C(s)] = 0 \\ Q_M&:\quad -\frac{\delta F}{\delta Q_M} + \lambda_A(1) + \lambda_A(2) + \frac{\delta F}{\delta Q_M}(1 + R_M)(\lambda_B(1) + \lambda_B(2)) = 0 \end{aligned} \qquad (2a)$$

also: $\emptyset_A(s)\lambda_A(s) = 0; \qquad \emptyset_B(s)\lambda_B(s) = 0.$

A variety of other regulatory strategies can also be analyzed by changing the constraints $\emptyset_B(s)$, but Eqs. (1a) seem to be the best simple representation of actual practice.

A Numerical Example

The implications of Eqs. (2a) turn out to be surprisingly complex and difficult to interpret. Therefore it will help to start with a numerical example. We assume that the firm is a monopoly, and faces the following contingent demand functions:

$$Q(1) = 2200 - 400\ P(1)$$

$$Q(2) = 3000 - 400\ P(2)$$

Thus:

$$P(1) = 5.5 - .0025\ Q(1)$$

$$P(2) = 7.5 - .0025\ Q(2)$$

Costs are also contingent on the state occurring: $C(1) = 1.5$ and $C(2) = 1.0$. The present investment required per unit of capacity is 1.0; i.e., $F(Q_M) = (1.0)\ Q_M$. Present values are obtained via the assumed prices $V(1) = .5$ and $V(2) = .4$. This implies a risk-free rate of interest of about 11 percent.

Using the conditions stated in Eqs. (2a) the optimal solution for $Q(1)$, $Q(2)$ and Q_M can be calculated. The values are shown in the first column of Table 1, along with several other characteristics of the solution. These may be compared with column two of the table, which shows the solution if the monopoly could be forced to behave like a perfect competitor.

The results for the competitive case were calculated from Eqs. (2), but with price substituted for marginal revenue in each state.

The differences between the monopoly and competitive solutions are clear from the first two columns of Table 1. Only a few comments are called for.

When the firm is free to act as a monopoly it restricts capacity and output, charges high prices, and earns a whopping profit. The profit is reflected in a high positive net present value. In the competitive case there is no such ex ante windfall gain, although profits are high ex post if state 1 occurs.

In this two-state model, a firm's risk may be simply described by the ratio of its payoffs in states 1 and 2 -- or, alternatively, by the ratio $(1 + R(2))/(1 + R(1))$, where $R(s)$ is the ex post rate of return in s. (In a state-preference framework, the riskier assets are those which pay off relatively more in states with low V's and less in states with high V's.) Thus we see from Table 1 that the monopoly solution for this example results in a safer firm than the competitive solution.

Incidentally, although the monopoly is safer in this instance, this is not a general result.

Now let us see how the firm reacts to a regulatory constraint

Table 1

Results for Numerical Example

	Monopoly Solution	Competitive Solution	Constraint on Maximum Rate of Return ($R^* = .20$)	
			Response A.	Response B
Q(1)	812	1622	900	1320
Q(2)	812	1622	1920	302
Q_M	812	1622	1920	1320
Req'd Investment	812	1622	1920	1320
Net Present Value	1480	0	$+12 \cong 0$	106
R(1), Expost Rate of Return in s = 1	1.47	-.555	.05	.20
R(2), Ex Post Rate of Return in s = 2	2.97	.945	.20	.20
P(1)	3.47	1.445	3.25	2.2
P(2)	5.47	3.445	2.7	6.75
$\frac{1+R(2)}{1+R(1)}$	1.6	4.37	1.14	1.0

of $R^* = .20$. There are a number of responses the firm might make. One would be to expand investment and output in $s = 2$ until $R(2) = .20$, and to maximize profits if state 1 occurs, subject of course to the constraints $\emptyset_A(1)$ and $\emptyset_B(1)$. The results for this case are shown in the third column, headed "Response A," in Table 1.

Given this response, the effect of the constraint is to drive the net present value of the utility from 1480 to approximately zero, i.e. to eliminate the ex ante monopoly profit. This presumably is a good thing. However, other characteristics of the constrained solution are not so desirable. The firm is led to invest more than it would in the competitive case; it does this to drive down the ex post rate of return in $s = 2$. This additional capacity is used only in state 2, however. The firm has invested enough so that the constraint on maximum profit is not binding in $s = 1$, and so the firm seeks maximum profit in that state, restricting output and driving up prices. Even at an output of 900, where $MP(1) = 0$, it can only earn 5 percent on the investment of 1920.

In short, the regulatory constraint would eliminate ex post "monopoly profits" in one state at the expense of allowing the firm freedom from regulation if the other state occurs.

It is also interesting that in this example the firm becomes safer from the investor's point of view when it operates under the constraint.

It must be emphasized that this is one of several ways in which the firm might react to imposition of a rate of return constraint. But I will defer consideration of the other responses to a bit later in the paper.

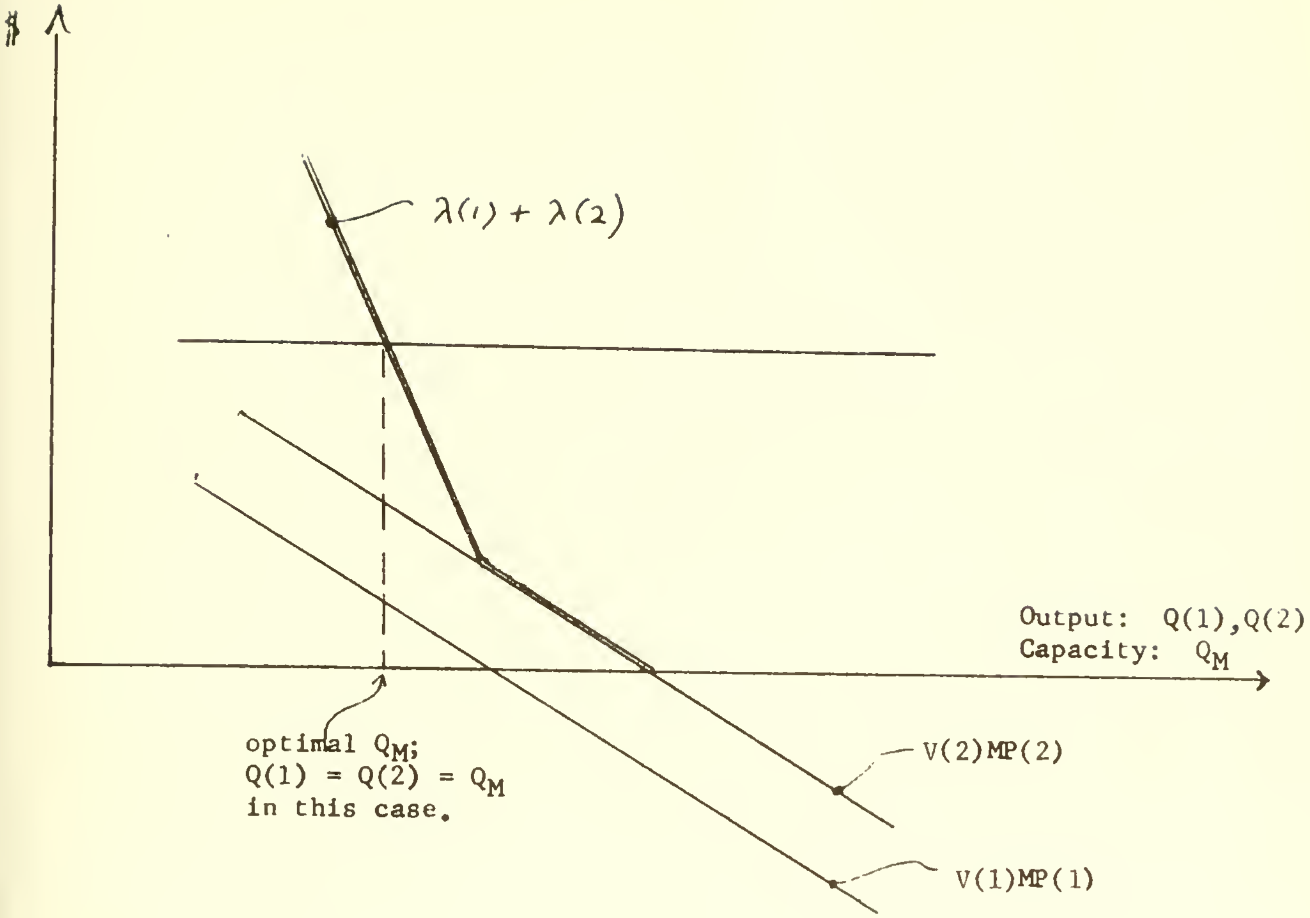
A Geometric Treatment of the Model

The consequences of the model just described can clarified by a geometric presentation. I will first present the solution for an unconstrained monopoly and then show how the firm can respond when a constraint is imposed on ex post rate of return.

For convenience, it will be assumed that $\delta F/\delta Q_M = f$, a constant. That is, the marginal investment required to add a unit of capacity is constant. Also, let $TP(s) = Q(s)[P(s) - C(s)]$, the total cash return realized in state s. Note that $MP(s) = \delta TP(s)/\delta Q(s)$.

Figure 1 shows the conditions for equilibrium for the unconstrained monopoly case. They are simply that f equals the sum $V(1)MP(1) + V(2)MP(2)$. This is the condition for Q_M in Eqs. (2a). Since f is a positive constant, MP(s) must be positive in at least one state. The monopoly will not build enough capacity to allow it to maximize TP(s) in all states.

This result is shown in Figure 2, which plots TP(s) for each state as a function of potential or actual output. In the case shown, both MP(1) and MP(2) are positive at the optimal Q_M, so the monopoly



Note: $\lambda(s) = \max(q(s)MP(s), 0)$.

FIG. 1

Conditions for Equilibrium -- Unconstrained Monopoly

will produce at capacity in each state. (This would be a frequent result in highly capital intensive industries.)

The solid ray extending from the origin in Figure 2 represents F, the amount of investment as a function of Q_M. Any constraint on ex post rate of return limits TP(1) and/or TP(2) to an amount proportional to F. Given R^*, we can thus represent the level of the constraint as a function of Q_M by a ray lying above F. The higher R_M, the higher the slope of the ray.[6] Given Q_M the rays Z, Y, X, W and V define a series of constraints resulting from successively higher values of R^*.

Once Q_M is established the constraint on total profits is independent of output in either state. Thus, if capacity is that called for by the monopoly solution and if R^* is set at level V, then the constraint on TP(1) and TP(2) may be represented by the dashed horizontal line shown in Figure 2.

Now we can see how the utility's output and investment decisions change if R^* is set at a high value (level V) but then gradually lowered. We continue to assume that the utility's response to a binding constraint is to lower price and increase output in the binding state.

Level W. -- At this level the monopoly solution violates the constraint in state 2. The utility is thus forced to increase investment Q_M. Since MP(1) and MP(2) are both positive at the monopoly solution and at the new levels of output and capacity, both output and "total profits" TP(s) increase in each state. The net present value of the firm declines, however.

Level X. -- As R^* is decreased to level X, the utility is

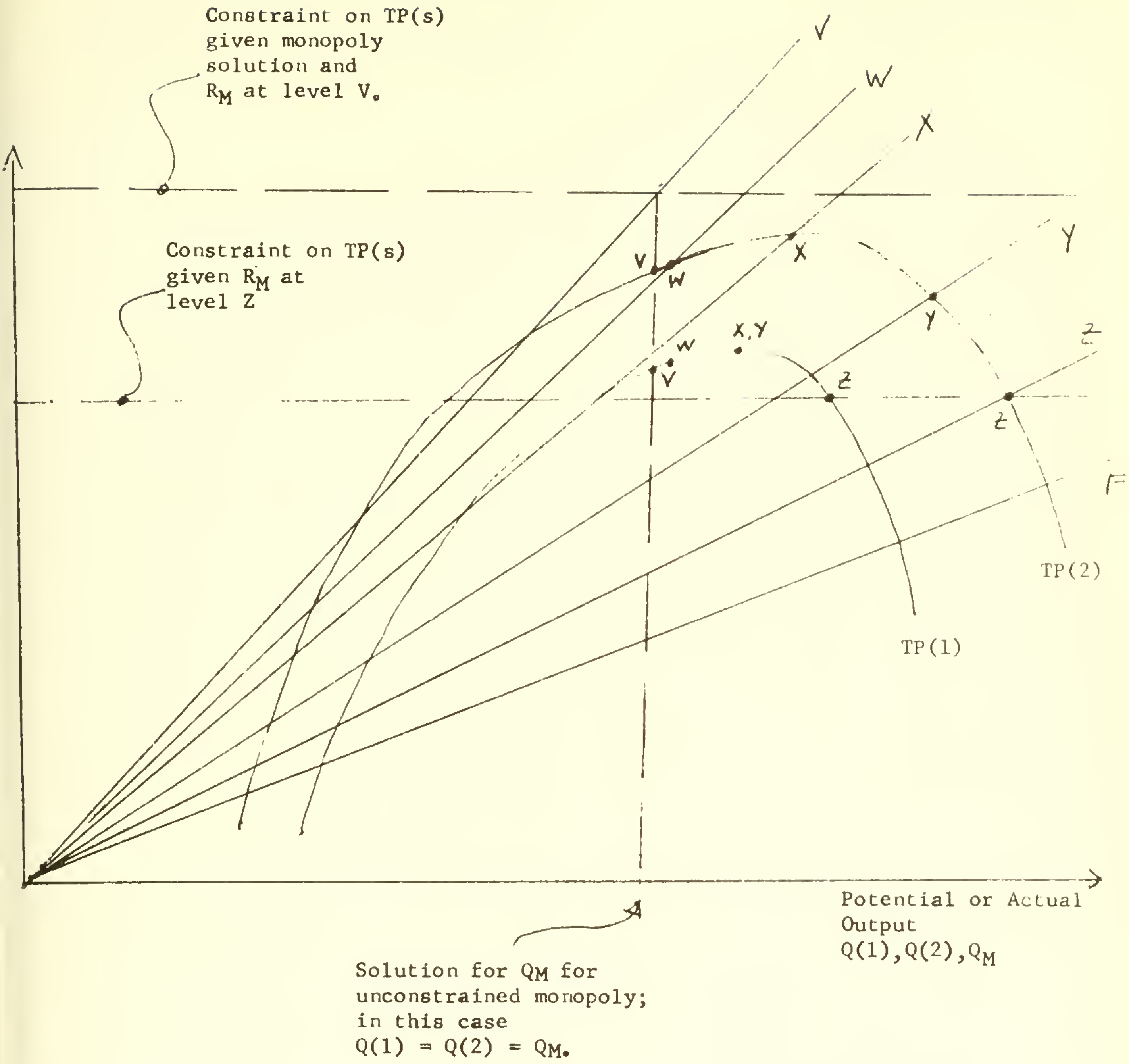

Constraint on TP(s)
given monopoly
solution and
R_M at level V.
V
W
X
Y
Z
F
Constraint on TP(s)
given R_M at
level Z
V
W
X
Y
Z
V
W
X, Y
Z
TP(2)
TP(1)
A
Potential or Actual
Output
Q(1),Q(2),Q_M
Solution for Q_M for
unconstrained monopoly;
in this case
Q(1) = Q(2) = Q_M.

FIG. 2

Effects of ex post Rate of Return Constraint

on a Utility's Output and Investment Decisions

forced to increase Q_M and Q(2) still further to satisfy the constraint in state 1. Q(1) will be increased also, but only up to the point at which TP(1) is maximized. There will be excess capacity in s = 1 because the constraint on ex post rate of return is not binding in s = 1, and because increasing output to $Q(1) = Q_M$ would reduce TP(1). In other words, the utility can act like an unconstrained monopolist in that state.

Level Y. -- Reducing R^* to level Y does not change Q(1), but forces the utility to increase Q(2) and Q_M. The effect is to decrease TP(2).

Level Z. -- Finally, at level Z, the constraint is binding in both periods. As R^* is moved from level Y to Z, both Q(1) and Q(2) are increased, although the utility will still have excess capacity in state 1. Since TP(1) = TP(2) the effect of the constraint at level Z is to make the utility an absolutely safe firm -- assuming, of course, that it is still willing to stay in business at this point.

Note that the results obtained for response A in Table 1 are are of type X or Y.

Effects of the Ex-Post Constraint on the Utility's Risk

One useful feature of the model as presented in Figure 2 is that it shows how regulation affects the risk of the utility as seen by its shareholders. In particular, we have just shown that if an ex post constraint on rate of return is set low enough, then it will either force the firm out of business, or make it safer than it would be if it were unregulated. It is as if the regulators were saying to the utility, "We'll forced to increase q_t and $q(t)$ still further to satisfy the constraint in which q [illegible] will be increased also, but only up to the point at which $T(1)$ is [illegible]. There will be excess capacity if $n > 1$ because the [illegible] is not binding if $n > 1$, and because the [illegible] q_t would reach $T(1)$. In other words, the utility [illegible] constrained [illegible] to that stretch [illegible] T decreases [illegible] (t), but forces [illegible] (t) and q_t. The effect is to decrease $T(1)$ [illegible]

[illegible]

make you a safe investment if it kills you."

There is more to the matter than that, however. A close look at Figure 2 shows the marginal effect of a small decrease in R^*, starting from each of the levels V, W, X, Y and Z. In this context, it is easiest to interpret "risk" as the ratio of TP (2) to TP (1). At V there is of course no change in risk. At W, the effect is ambiguous, since both TP(1) and TP(2) are affected. At X risk increases, since TP(2) increases as R^* declines, but TP(1) is unchanged. Finally, risk decreases at levels Y and Z, since TP(2) decreases and TP(1) is either constant or equal to TP(2). In short, the conclusion is that risk may actually increase if R^* is decreased from relatively generous levels. Ultimately, however, a decrease in R^* makes the utility safer, if it is in business at all.

Other Responses to the Regulatory Constraint

All this has assumed that the firm responds to the rate of return constraint by investing more and producing more in the high-profit state. However, there are other responses that may be better from the firm's point of view.

Figure 3 is Figure 2 redrawn, except that the effects of the rate of return constraint are shown only for level Y. The type of response discussed so far will lead to the investment and output decisions labelled "A."

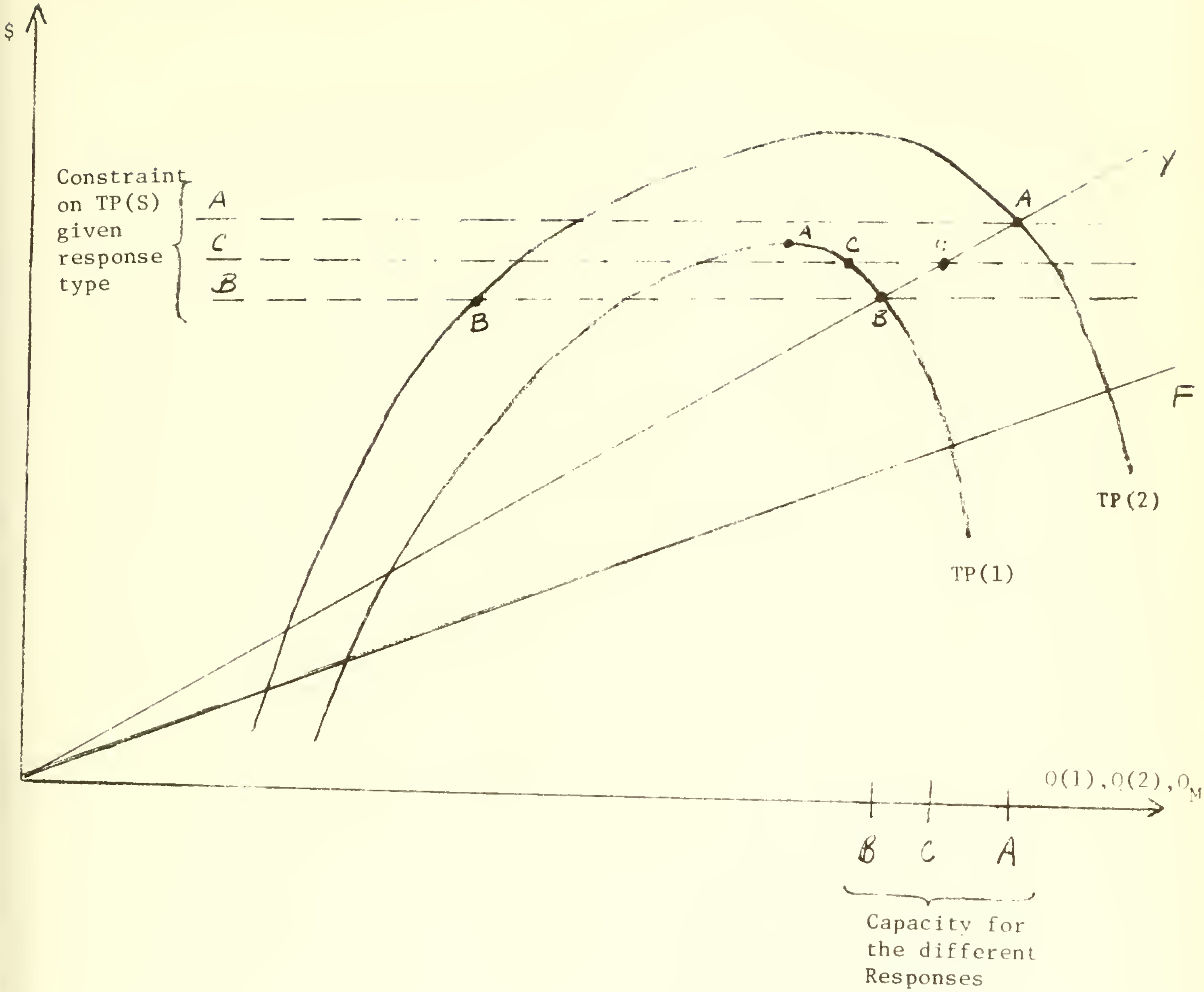

$
Constraint on TP(S) given response type
A
C
B
A
C
B
A
C
A
B
Y
F
TP(2)
TP(1)
Q(1),Q(2),Q_M
B
C
A
Capacity for the different Responses

Fig. 3

Alternative Responses to Rate of Return Constraint

Actually, the firm may be better off taking "Response B." Here the firm invests and produces only enough to satisfy the rate of return constraint in state 1, the "low-profitability" state. Given this lower level of investment the firm could earn substantially more than the minimum return R^* if it produced at capacity in s = 2 and did not ration its product. To satisfy the constraint in s = 2 without rationing it increases price, reducing demand further and further below capacity, until finally the constraint is satisfied.

Now turn to the fourth column of Table 1 which shows the results for the numerical example, given response B. Compared to response A, we find that although investment is lower, the firms net present value is increased. The increase occurs because the firm now earns the maximum 20 percent return in each state rather than in state 2 only; the present value of the decrease in TP(2) is more than offset by the increase in TP(1) and the reduced investment

Of course, strategy B will not always be prefereble to A. The choice between them would depend on R^*, demand and cost conditions, etc.

If the firm is allowed to ration its output in some states, still another response is available. The firm could, for example, produce at B in s = 2, and it could produce the same quantity in s = 1 while still keeping price low enough to satisfy the rate of

return constraint. The excess demand in $s = 1$ would be "cured" by rationing. The firm's net present value would be the same as under response B.

Still better, the firm could increase capacity -- say to C -- allowing it to produce and earn more in each state. It will increase investment from level B until the incremental return on investment drops to the risk-free rate.

Although the *opportunity* to ration will never decrease a firm's net present value, it does not follow that rationing will *always* be a desirable strategy for the constrained firm.

Implications. -- It may be argued that cases B and C are not of practical interest. Responses like B would lead utilities to raise price and produce less relative to capacity in response to an unexpected upward shift in demand -- which is exactly opposite to how the actual process of regulation works. Similarly, it is not at all clear that rationing is a tolerable response to regulation. Naturally, it occurs from time to time when a mistake is made and capacity falls short of demand. But rationing in these instances is accepted as an inescapable short-run evil -- remember the public outcry when telephone service was rationed in New York City and elsewhere?

Thus responses of type A seem to be of the most practical interest. Nevertheless, the fact that there are a variety of strategies open to the firm even in this simple case, and the fact that the firm could thereby be enticed into rather odd behavior, leave one a bit worried about the possible effects of regulation in the face of uncertainty and the complications of real life.

Further Theorems and Conclusions

The main conclusion of this paper is probably clear by now, but it should nevertheless be stated precisely and proved.

Consider a monopolistic firm in the one-period world I have assumed. The object of regulation is to find a value for R^* such that the firm will act as if it were in perfectly conpetitive markets. This "competitive solution" will be denoted by $\hat{Q}_M$, $\hat{Q}(s)$.

Assume there is uncertainty about demand and/or cost conditions, and that the firm's profits are uncertain at the competitive solution $\hat{Q}_M$, $\hat{Q}(s)$.

Is there always some value of R^* that would lead to $\hat{Q}(s)$, $\hat{Q}_M$? Unfortunately, the answer is no.

Suppose the firm has somehow arrived at a provisional investment and output plan which calls for $\hat{Q}(s)$, $\hat{Q}_M$. It clearly will not stick with this plan once it considers the alternatives. However, is there some rate of return constraint which will prevent the firm from deserting the competitive solution?

If such a constraint exists then R^* must be exactly equal to $\hat{R}(z)$

[illegible] of [illegible] of the most [illegible] terest. However, the fact that [illegible] a variety of [illegible] open to [illegible] in [illegible] and the fact that the [illegible] could [illegible] and [illegible] about the [illegible] for [illegible] the [illegible]

Further [illegible]

[illegible]

It should [illegible]

[illegible] meeting [illegible]

the [illegible]

[illegible]

where $\hat{R}(z)$ is the highest rate of return that might be achieved in the competitive case. This means that the rate of return constraint will not be binding in the other states.[7]

Consider a state s from among these others. Clearly $\hat{Q}(s)$ must be less than or equal to $\hat{Q}_M$. Suppose it is less; then at $\hat{Q}(s)$, price equals MC(s), MR(s) < MC(s), and there will be an incentive for the firm to raise price and reduce Q(s). Since the constraint on rate of return is not binding, the constraint cannot maintain the competitive solution.

However, it is possible that $\hat{Q}(s) = \hat{Q}_M$ for all s, and that the firm would have no incentive to change price or output in any state. This would require MR(s) > MC(s) in all states when output is at the capacity $\hat{Q}_M$.

There is, of course, an incentive to reduce $\hat{Q}_M$. However, the firm cannot do so if $\hat{Q}(z) = \hat{Q}_M$, since setting $Q_M < \hat{Q}_M$ would require $Q(z) < \hat{Q}(z)$, and this would lead to a violation of the rate of return constraint. Note that the condition $\hat{Q}(z) = \hat{Q}_M$ requires that $P(z) \geq MC(z)$ at the output $\hat{Q}(z)$.

To sum up, imposing the competitive solution $\hat{Q}(s)$, $\hat{Q}_M$ on a monopolistic firm requires setting $R^* = \hat{R}(z)$ and the existence of two conditions. The first is that $\hat{Q}(z) = \hat{Q}_M$; i.e., that price is greater than or equal to MC(z) at the output $\hat{Q}(z)$. The second is that MR(s) > MC(s) at the outputs $\hat{Q}(s)$ for all $s \neq z$. This implies that $\hat{Q}(s) = \hat{Q}_M$, although the converse does not hold. Neither condition is generally true, although admittedly each is more likely to be true, the more highly capital-intensive the industry.

FOOTNOTES

[*] Associate Professor of Finance, Sloan School of Management, Massachusetts Institute of Technology.

The ideas in this paper have been presented at two conferences. The first was sponsored by the Inter-University Committee on Public Utility Economics and held at Michigan State University in February 1971. The second was the AT&T Conference on Financial Aspects of Utility Regulation, held at Stanford University in July 1971. My audiences at these conferences supplied many helpful comments.

Mr. Stavros Thomadakis deserves special thanks for his detailed comments and advice on the paper.

1. Federal Power Commission et. al. vs. Hope Natural Gas Company (1949). 320 U.S. at 603.

2. Wilcox vs. Consolidated Gas Company. 212 U.S. 19, Bluefield Water Works and Improvement Company vs. Public Service Commission, 262 U.S. 679.

3. For a detailed exposition see J. Hirshleifer, Investment, Interest and Capital (Englewood Cliffs, New Jersey, 1970).

4. A model similar to this one was arrived at independently by S. C. Littlechild in "A State-Preference Approach to Public Utility Pricing and Investment Under Risk." (Unpublished ms., Graduate Center for Management Studies, Birmingham, England.) However, Littlechild does not investigate the effects of regulation.

However, this is not a very comfortable result even if the conditions really are true. Since R^* would be set at the highest possible competitive rate of return the rate of return constraint would be binding ex post only a small percentage of the time. A utility could violate it ex ante with confidence that it would be discovered only rarely. If the regulatory authorities reduced R^* to "catch" the utility more frequently, it would drive the utility to the type of behavior illustrated by Responses A, B and C in Table 1 and Fig. 3. There would seem to be no practical way to use a straight-forward rate of return constraint to enforce competitive behavior under uncertainty.

5. This assumes, of course, that the required subsidy is less than the consumer surplus generated at the competitive solution.

6. The constraints shown are linear because F is assumed to be a linear function of Q_M. This is not necessary to the argument, but it makes a neater diagram.

7. This assumes that the maximum competitive rate of return occurs in only one state, i.e. that $\hat{R}(s) < \hat{R}(z)$ for all $s \neq z$. It is conceivable that there will be some state y in which $\hat{R}(y) = \hat{R}(z)$. However, the discussion in the text can be applied to y as well as to z.

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